Reinforced concrete corrosion: application of Bayesian networks to the risk management of cooling towers in nuclear plants

B. Capra & J. Le Drogo
OXAND S.A., Avon, France

ABSTRACT: Authors of papers to proceedings have to type these in a form suitable for direct reproduction by the publisher. Degradation modelling of concrete structures uses uncertain variables and leads, using reliability assessment, to time dependant evolution of failure probabilities. However, only few data are generally available to feed models leading to two types of uncertainties: an intrinsic one depending on the modelled phenomena and one related to the precision of the measurements. Each new data available is a piece of information which allows updating the initial prediction. In this article, an example of updating process, based on a Bayesian network, is presented and applied on the corrosion risk of a cooling tower in a nuclear plant.

1 INTRODUCTION

Risks assessment associated to assets ageing, and in particular to the ageing of civil engineering infrastructure is an important stake for the future. Today, the technical challenge is not only to build new infrastructure but to maintain those existing because economic stakes are considerable. In this context, OXAND develops advanced solutions to assess risks associated to infrastructures ageing. These technologies allow exploiting instrumentation and inspection data in order to define optimized maintenance strategies. The benefit for the structure owners are better risk management and lower maintenance budgets. In order to anticipate and optimize these costs, it is necessary to have representative ageing models and the most reliable input data as possible.

2 CONTEXT

Cooling towers, like other concrete structures, are subjected to time dependant environmental effects (wetting/drying cycles, temperature and humidity gradients …). Among these different solicitations, carbonation-based corrosion is pathology of great importance which can lead to a decrease of mechanical performance of the structure. The carbonation and corrosion process evolution can be forecast using more or less physical models leading to an estimation of the rate of ageing (figs 1-2).

3 MATERIAL DATA ACQUISITION

Some core samples were extracted from a 25 years old cooling tower and tested in order to determine the compressive strength of the concrete Rc and the depth of carbonation X. The experimental data collected are summarized in table 1.

These data are based on destructive tests which require core samples, sometimes difficult to obtain on such structures, leading to limited amount of data for non negligible cost. For this reason, the statistics showed on table 2 might be use very carefully because they are limited in number and an incorrect measure can skew them.

Material properties are not the only uncertain parameters. During the building process of the cooling tower, concrete cover thickness is also a parameter subjected to random variations. On site measurements, made using a pachymeter, were approximated by the following probability density function of concrete cover d (fig. 3).

The relative humidity of concrete in such a structure is strongly related to environmental conditions. We make the assumption that this parameter is constant with a representative mean value of 75% for the considered cooling tower environment.

Figure 1. Cooling tower

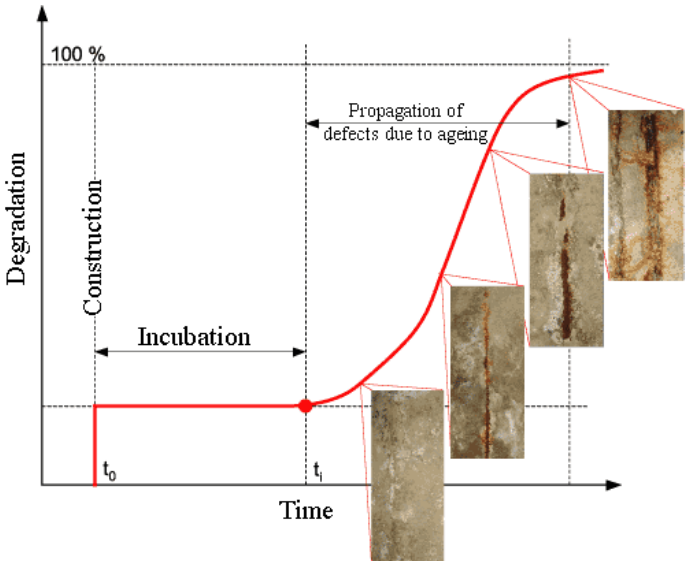


Figure 2. Concrete degradation vs. time.

Table 1. Concrete strength and carbonation depth measured at 25 years

Rc (MPa)	39.1	46.1	52.0	49.2	48.3	52.8	48.8	53.6	44.2	54.1
X (mm)	4.6	5.2	6	6.6	12	3	6.4			

Table 2. Statistics based on field measures

	Mean	Standard deviation	Range
Rc(MPa)	48.8	4.7	15
X (mm)	6.3	2.8	9

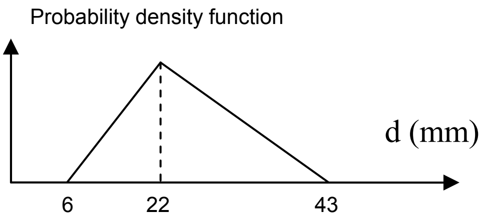


Figure 3. Probability density function modelling the variability of concrete cover d.

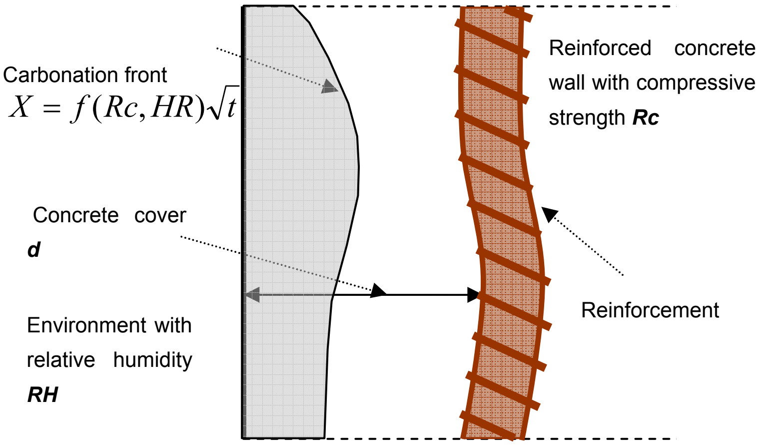


Figure 4. Carbonation front propagation

Finally, an exhaustive visual inspection of the cover leaded to a measure of the apparent cracking length related to corrosion. Indeed, steel corrosion forms corrosion products which take more volume than the initial steel. This volume change exerts pressures which are able to exceed tensile strength of concrete leading to cover cracking. This measurement is not fully representative of the total length of corroded steel because the initiation period, during which corrosion products are not able to crack concrete, is missing. During this inspection, the cumulated length of apparent cracked concrete related to corrosion was 115 m for an overall length for external steels of approximately 500 km. This gives an order of magnitude of 10^{-4} for the apparent corroded steel proportion. By consequence, the real proportion of corroded reinforcement is thus higher than 10^{-4} after 25 years.

4 DETERMINISTIC STUDY

In this article, a phenomenological carbonation model (see figure 4) is used to predict carbonation front depth X. This model takes into account concrete compressive strength Rc, environment relative humidity RH and the time t (Petre-Lazar 2000).

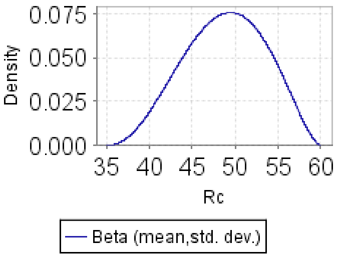


Figure 5. Concrete strength Rc distribution law (Beta).

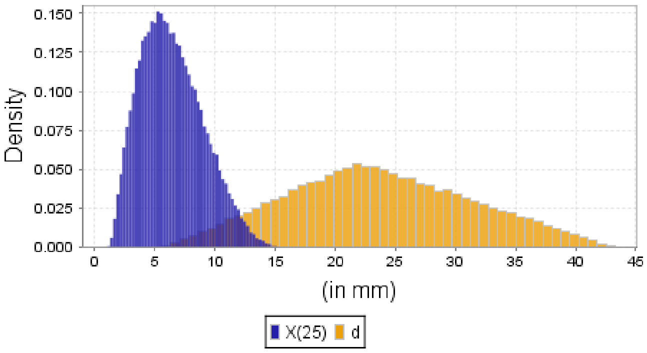


Figure 6. Probability density functions of calculated X at 25 years, X(25), and of concrete cover d.

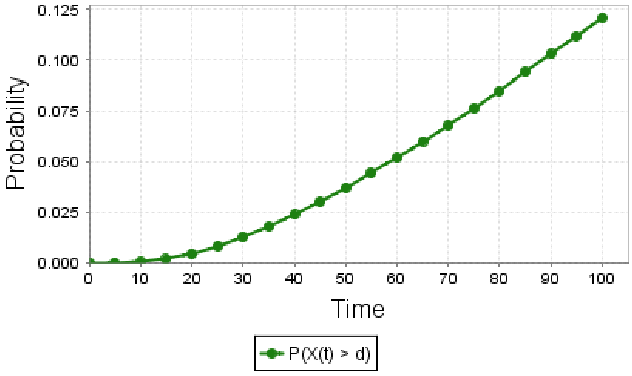


Figure 7. Probability of corrosion initiation versus time, P(X(t) > d).

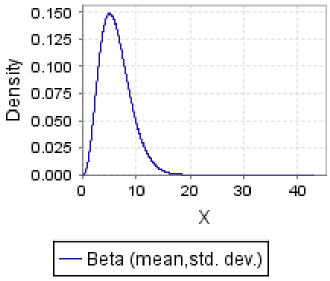


Figure 8. Carbonation depth X distribution law (Beta).

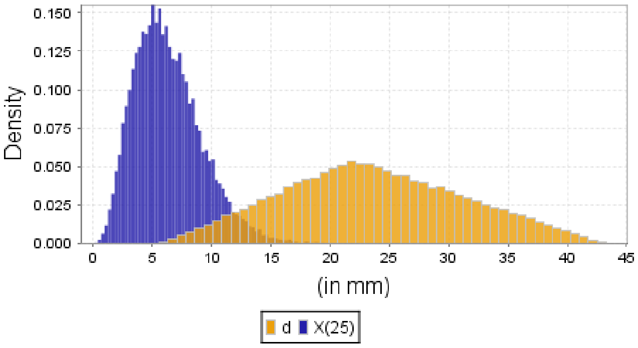


Figure 9. Probability density functions of X at 25 years, X(25), and of concrete cover d.

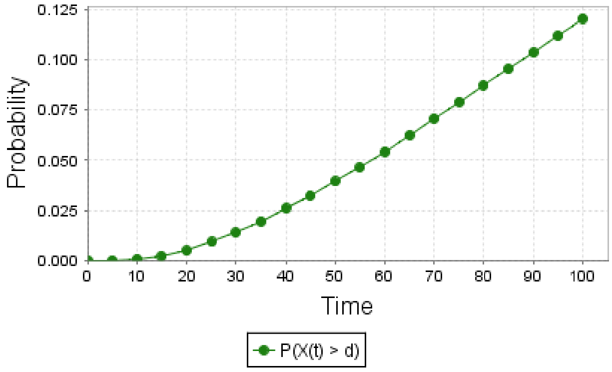


Figure 10. Probability of corrosion initiation versus time, P(X(t) > d).

It is considered that corrosion process begins when the carbonation front reaches reinforcement.Using mean values of the different parameters of the problem (Rc, RH) leads to a carbonation front of 6.2 mm at 25 years. If we consider the mean value of measured concrete cover, 22 mm, the corrosion might not have initiate. Thus, a deterministic study can not explain the on-site observed corrosion. In order to explain the corrosion-related cracking, a probabilistic study has been developed.

5 PROBABILISTIC STUDY

The statistics of the previous section can be used in a probabilistic study but we have to keep in mind that they are not complete because related to only few data. In this section, we will consider that relative humidity RH is not a random variable because a sensitivity analysis of the model showed that this parameter is less sensible than the others.

We model concrete compressive strength Rc distribution by a beta distribution of mean value 48.8 MPa, standard deviation 4.7 MPa, minimum 35 MPa and maximum 60 MPa (fig. 5). The probability density function of X, calculated by the model at 25 years, X(25), is shown on figure 6 and compare to concrete cover d distribution The related probability of corrosion initiation, P(X(t) > d), is shown on figure 7.

Instead of using measurements on concrete compressive strength Rc, we can also use those directly available on carbonation depth X. We choose a beta distribution of mean value 6.3 mm, standard deviation 2.8 mm, minimum 0 and maximum 43 mm (fig. 8). The probability density function of X at 25 years, X(25), is shown on figure 9 and compare to concrete cover d distribution The related probability of corrosion initiation, P(X(t) > d), is shown on figure 10. The two modelling (calculated or measured X) lead to similar corrosion initiation probabilities versus time which means that the carbonation model used is suitable for this study. These two approaches give close results but all the data are not used at the same time, so the available information is not fully used.

The deterministic approach, based on the mean value of all the parameters, was not able to represent the apparent corroded reinforcement fraction (10^{-4} at 25 years). The probability of corrosion initiation obtained by the probabilistic approach give a value of 10^{-2} at 25 years. This value is higher because it represents the total length of corroded reinforcement and not only those which create and observable crack at the concrete surface. In order to compute the apparent corroded length, it would be necessary to determine the steel section loss able to crack concrete which is not the purpose of this article.

6 BAYE’S THEOREM AND BAYESIAN NETWORK

6.1 *Baye’s theorem*

The classical probabilistic approach is able to explain the apparent corroded length observed by taking into account uncertainties but is not able to consider all the available data at the same time (Rc or X). Bayesian approaches are tools which allow such an approach.

A simple example of Bayes’ theorem is presented on figure 11. We consider the production of red and white pieces by three machines (M1, M2 and M3). We assume the number of red and white pieces produced by each machine (100% for M1, 50% for M2 and 25% for M3).

Before having more information, we can define the prior probabilities P. If someone picks a piece at random, the probabilities to pick a piece whose colour C is red or white are equals to 0.5: P(C=red)=0.5 and P(C=white)=0.5.

In the same way, the probability to pick a piece from machine 1 or 2 is equal to 0.25 and 0.5 for machine 3: P(M1)=0.25; P(M2)=0.25 and P(M3) = 0.5.

Now, if we pick a piece whose colour is red, then the Bayes’ theorem allows to calculate the posterior probabilities for the piece to come from machine 1, 2 or 3 (P(Mi | C=red). Bayes’ classic theorem is used to calculate this conditional probability:

$$P(M_i|C) = \frac{P(C|M_i)P(M_i)}{P(C)} = \frac{P(C|M_i)P(M_i)}{\sum_i P(C|M_i)P(M_i)} \tag{1}$$

Here, for example, the probability that the red piece comes from machine 1 is 0.5 and 0.25 for machines 2 and 3:

$$\begin{aligned} P(M1|C = red) &= \frac{P(C = red|M1)P(M1)}{\sum_i P(C = red|M_i)P(M_i)} \\ &= \frac{1 * 0.25}{1 * 0.25 + 0.5 * 0.25 + 0.25 * 0.5} = 0.5 \end{aligned} \tag{2}$$

$$\begin{aligned} P(M2|C = red) &= \frac{P(C = red|M2)P(M2)}{\sum_i P(C = red|M_i)P(M_i)} \\ &= \frac{0.5 * 0.25}{1 * 0.25 + 0.5 * 0.25 + 0.25 * 0.5} = 0.25 \end{aligned} \tag{3}$$

$$P(M3|C=red)=\frac{P(C=red|M3)P(M3)}{\sum_i P(C=red|M_i)P(M_i)} \tag{4}$$
$$=\frac{0.25*0.5}{1*0.25+0.5*0.25+0.25*0.5}=0.25$$

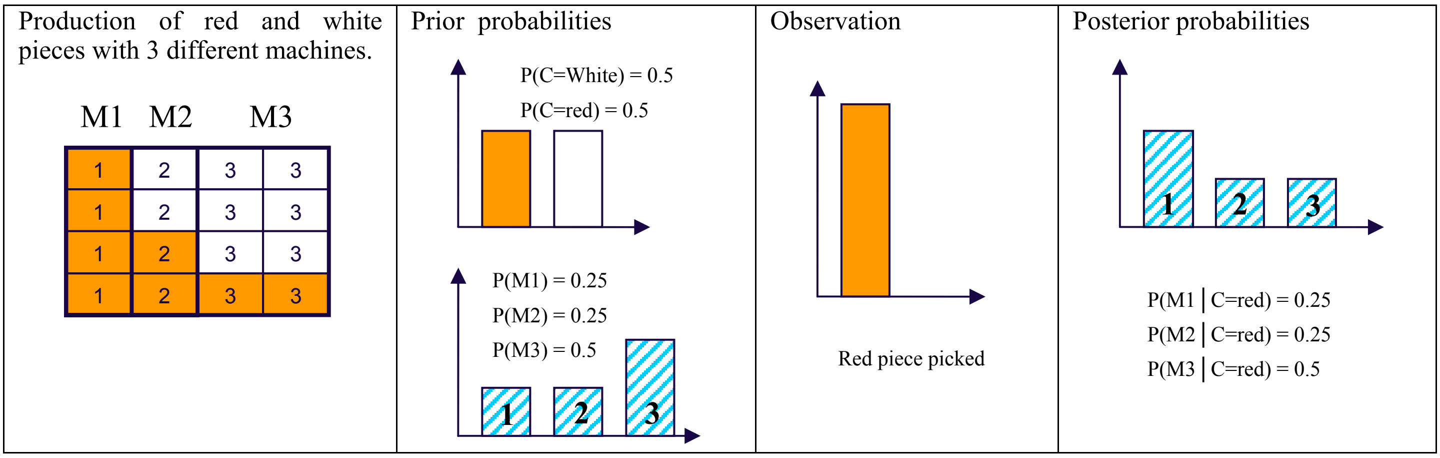


Figure 11. Basic example of Bayes' theorem

This example uses discrete variables and calculation can be done easily. It is also possible to use continuous variables rather than discrete ones. Instead of using probabilities, the Bayes' theorem gives a relation between probability density functions fx:

$$f_X(x|Y=y)=\frac{f_Y(y|X=x)f_X(x)}{f_Y(y)} \tag{5}$$
$$=\frac{f_Y(y|X=x)f_X(x)}{\int_{D_X} f_Y(y|X=x)f_X(x)dx}$$

The calculation of the posterior law is much more difficult in the continuous case. For some laws, like conjugate prior, this can be done easily but, in the general case, the only solution is often to use simulation algorithm (Droesbeke et al. 2002)(Procaccia & Suhner 2003).

6.2 *Bayesian network*

A Bayesian network is a directed acyclic graph where nodes represent variables and arcs represent dependence relations among these variables.

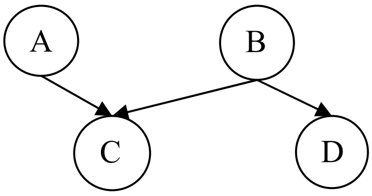


Figure 12. Example of a Bayesian network

A Bayesian network is a representation of the joint distribution over all the variables represented by nodes in the graph. For the network of figure 12, the joint distribution P(A,B,C,D) can be decomposed as follows:

$$P(A, B, C, D) = P(A).P(B).P(C|A, B).P(D|B) \tag{6}$$

When an observation, or evidence, is put on one node, inference algorithm produces a conditional distribution of the variables given the evidence: the posterior distribution. This inference relies on Bayes' theorem.

Oxand has developed a software for the simulation of Bayesian networks, SIMEO MC², using a Gibbs Sampling algorithm which belongs to the family of Markov Chain Monte Carlo (MCMC) algorithms. A MCMC method is an algorithm for sampling from probability distributions based on the construction of a Markov chain which has the desired distribution as its stationary distribution. A large number of draws leads to a sample from the desired distribution. To check the convergence of such an algorithm, the software allows superimposing convergence curves (curve of statistic, function of the draws number) of multiple simulations. To quantify the residual error, statistics should be done on several simulations. For more information about MCMC algorithm, see (Gilks 1995)(Vierra 1999)(Billy et al. 2003)(Grenager 2004).

7 APPLICATION OF BAYESIAN NETWORKS

Using the previous carbonation model, we construct a Bayesian network able to integrate the different available measures (figure 13).

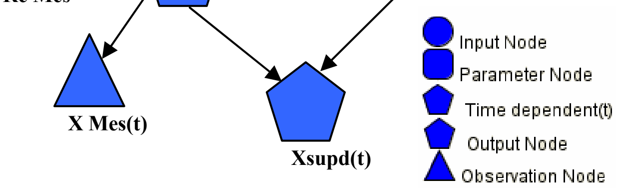


Figure 13. Carbonation bayesian network.

7.1 *Conditional and prior laws choice*

We choose to model the Rc variable by a conditional triangle law with mean value Rc_P0 and base Rc_P1. In a Bayesian network, conditional law parameters can also be random variables. These parameters will be updated when adding observations on the other nodes of the network.

Rc_P0 and Rc_P1 need a prior law. First, we test an uninformative prior using a large uniform law. In a second case, we test a more informative prior using triangle law. Lastly, we test an informative law for Rc_P0 which is in contradiction with the measures.

Table 3: Field measures based statistics

Case	Rc_P0 (Rc mean)	Rc_P1 (Rc base)
Uninformative prior	Uniform law: Range: [30 MPa, 60 MPa]	Uniform law: Range: [1 MPa, 30 MPa]
Informative prior 1	Isocel Triangle law: Mean: 45 MPa Range: [35 MPa, 55 MPa]	Isocel Triangle law: Mean: 20 MPa Range: [10 MPa, 30 MPa]
Informative prior 2	Isocel Triangle law: Mean: 40 MPa Range: [35 MPa, 45 MPa]	Isocel Triangle law: Mean: 20 MPa Range: [10 MPa, 30 MPa]

7.2 *Measures uncertainties*

For the concrete compressive strength, destructive measures are done using core samples. The part of uncertainty related to measure represents about 5 to 10 % of the obtained value. This uncertainty does not take into account the intrinsic spatial heterogeneity of concrete on the tower. For the carbonation depth, measures are done using a coloured indicator and a ruler. The uncertainty of such a measure is about 1 mm.

7.3 *Results*

The aim of these simulations is to test the prior influence on posterior results. For each corrosion initiation probability result, a reference result with round marks is taken from the previous probabilistic study (§3 – fig. 7). The computed results from Bayesian network, taking into account the different cases without measures (Fig. 14) and with data on Rc and X (Fig. 15), are reported with rectangle marks.

Figure 14 shows prior prediction of corrosion initiation probability without taking into account measurements. We can see that an uninformative prior leads to a higher probability compared to the probabilistic study which took into account measures on Rc only. Results with a more informative prior really depend on the quality of the expert prior. The first informative prior leads also to a higher probability than the probabilistic study. We can notice that the difference is less important than the uninformative prior. The second informative prior, in contradiction with future measures, leads to a really higher probability of corrosion initiation.

On figure 15, when both measures on Rc and X are integrated, which is only possible within Bayesian network and not in probabilistic study, results with uninformative prior, or first informative prior, are similar. This is a particular result because in this case, the two probabilistic results taking into account Rc or X, were very close leading to a few difference with Bayesian network which take into account the two random variables at the same time. In addition, prior information and measures go the same way leading to an important weigh of measures on the updated result.

At the opposite, for the second informative prior, Bayesian integration leads to a compromise between prior information and measures. We can see on figure 16 that the posterior density function of the concrete compressive strength mean parameter is concentrated at the right boundary of the prior range. In this example, we can notice incoherence between prior information and measures. In this case, prior information and measurement confidence would have to be reconsidered.

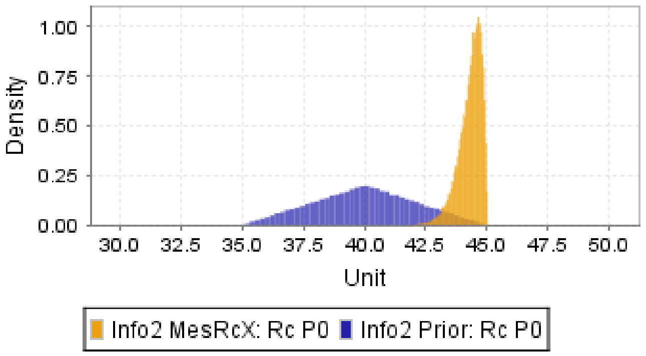
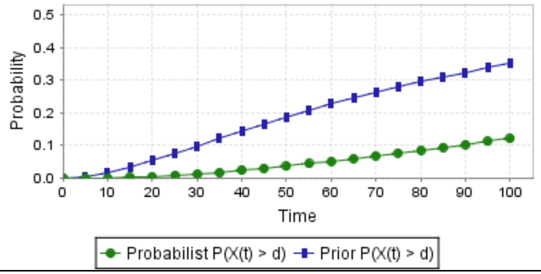
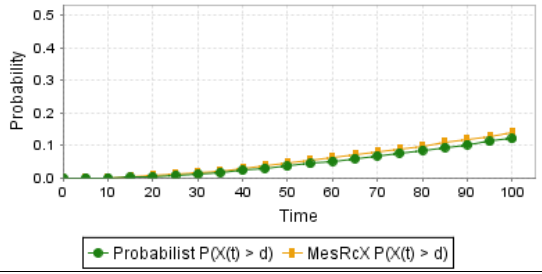
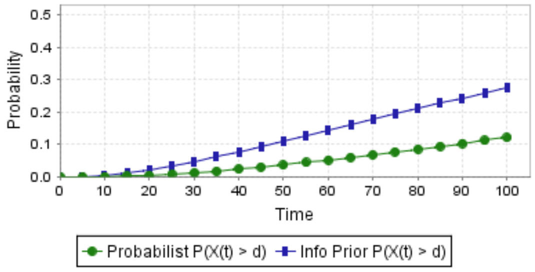
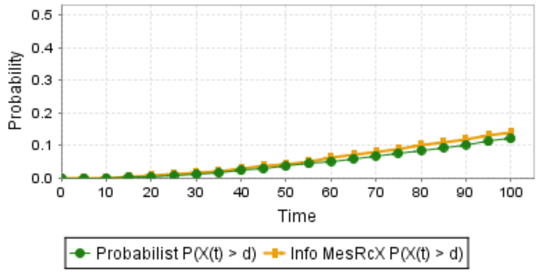
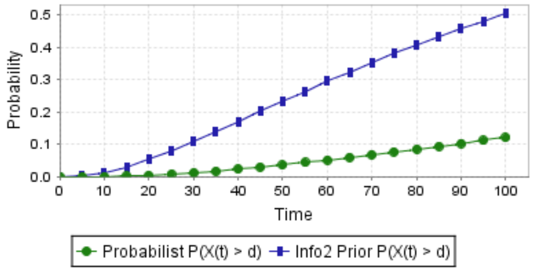
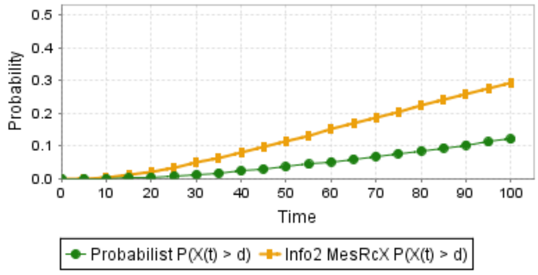


Figure 16: concrete compressive strength mean parameter prior and posterior probability density functions

Bayesian integration uses both concrete compressive strength measures and carbonation measures to update concrete compression strength knowledge. Bayesian updating reduces uncertainty on the concrete compressive strength integrating measures.

The prediction for carbonation is then less uncertain. Figure 17 illustrates this uncertainty evolution. Probability density functions after Bayesian updating are less spread. In this case, we can see that posterior density function of Rc is well contained in the prior range meaning that prior information and measures are coherent.

- User control of the confidence for the various sources of information. Contradictions between prior and measures can easily be detected observing prior and posterior parameters distributions;
- The graph of the model is easily understandable and usable for communication purpose.

	No available measurement	Measurements on X and Rc
Uninformative prior		
Informative prior 1		
Informative prior 2		

Figures 14: Probabilities of corrosion initiation with no available measurement

Figures 15: Probabilities of corrosion initiation with measurements on X and Rc

8 COMPARISON WITH PROBABILISTIC STUDY

Within the classical probabilistic study, it was only possible to use one type of the measures available (X calculated by measures on Rc or X measured directly). The two types of measures can not be used together leading to a loss of information because not all the data available are used. Bayesian networks offer great possibilities:

- Uncertain or indirect measures can be used for updating;
- Heterogeneous sources of information can be used (expert opinion, experience feedback…);

9 CONCLUSION

The application developed in this article shows that Bayesian networks allow a better control of uncertainty combining heterogeneous information like expert statements and uncertain ground measures. If few measures are available, the utility of such a method is undeniable.

Bayesian networks make possible for the engineer to balance relative confidences between expert opinion and measurements. This advantage is also one of the major difficulties of this approach as the results can be rather sensitive to these confidences. Thus, a sensitive study of the formulated assumptions might be included in Bayesian inference study.

Finally, we must keep in mind that if the results obtained by Bayesian networks bring additional elements compared to the traditional probabilistic analyses, interpretation must be done with the same care.

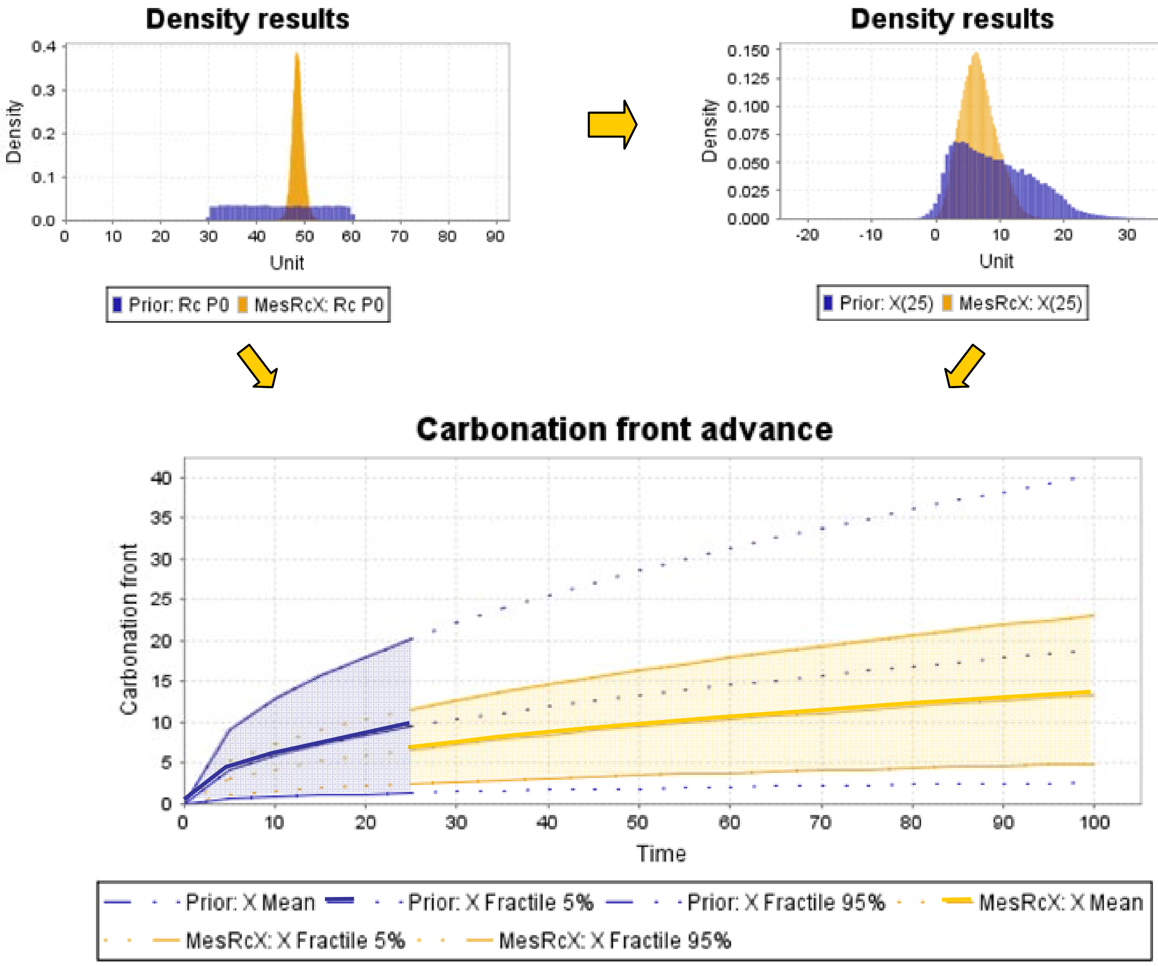


Figure 17: Updating of probability density functions and carbonation front depth.

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